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3. The annual dues of each institutional member shall be Seven Dollars (\$7), including two (2) subscriptions to the official journal.

4. All dues shall be payable on the first of January of each year. Should the annual dues of any member remain unpaid beyond a reasonable time, his name shall be dropped from the list, after due notice.

5. New members entering the Association after April 1 of any year shall have their dues pro rated for the balance of the year, except when they desire to receive the full current volume of the official journal.

ARTICLE VIII—AMENDMENTS TO THE ARTICLES OF ASSOCIATION AND BY-LAWS.

1. The Articles of Association may be changed to provide for a Board of Nineteen (19) Trustees by vote of a majority of the Three (3) Trustees named in the original Articles. Subsequent changes, amendments or modifications, of the Articles of Association and any amendments to the By-Laws may be made at any annual meeting of the Association, or at any adjourned session thereof, or at any special meeting of the Association called for such purpose, by a two-thirds (2/3) vote of those present and entitled to vote; *provided* that such amendment shall have been printed in the official journal at least one (1) month before the date of such meeting.

2. No change in the Articles of Association shall have legal effect until a Certificate thereof, verified by oath of the President and under Seal of the Association, attested by the Secretary-Treasurer, shall be filed in the office of the Secretary of State of the State of Illinois and recorded in the office of the Recorder of Deeds for Cook County, Illinois.

ARTICLE IX—INTERPRETATION.

1. "Board," wherever used in these By-Laws, shall be taken to mean a Board of Trustees consisting of the Officers, the Committee on Publications and the Trustees elected as such, as provided in Article III.

2. It being the intent that this Association continue as successor to the unincorporated The Mathematical Association of America, these By-Laws shall be construed liberally to that effect.

THE UNDERSIGNED, CHAIRMAN OF THE ORGANIZATION MEETING OF THE BOARD OF TRUSTEES OF THE MATHEMATICAL ASSOCIATION OF AMERICA (INCORPORATED), HEREBY CERTIFIES that the above is a full, true and correct copy of the code duly adopted at such meeting, held September 10, 1920, as the By-Laws of said Association.

WITNESS MY HAND AND THE CORPORATE SEAL, this 11th day of September, A. D. 1920.

[CORPORATE SEAL]

HERBERT E. SLAUGHT,

Chairman as aforesaid.

THE "DANGER AREA" CURVE.

By A. S. MERRILL, University of Montana.

A moving object (ship) is a target for a projectile (torpedo) which travels horizontally under its own power. Certain limiting conditions under which the projectile would be fired, or launched, are known. It is desired to determine the boundary curve of the area of probable positions of the firing agent relative to the target, at the time of firing.

Let the target, of length AB but of negligible width, be considered first as fixed in position. For certain known conditions of probable error, the projectile has a range R if fired from a point in the perpendicular bisector of AB . Construct a circle of diameter R through A and B (Fig. 1). Suppose AB to be small in comparison with R . The middle point of AB is then approximately at M , the end point of the diameter ML perpendicular to AB ,—sufficiently close to be

so considered for our problem. If the target is viewed from any other point upon the circle, as C , it is evident that it subtends an angle of the same size as that which it subtends when viewed from L . The probability of a hit is then the same for a projectile launched from any point upon the circle. In other words, the circle is the locus of points from which the projectile may be fired with the same probability of success for the fixed limiting conditions of probable error, and is the boundary curve of the area from which the projectile may be fired within the limits of probable success.

If now the target is considered as moving, there arise very difficult complications in the matter of aiming the projectile. With these, however, we are not at present concerned. For our problem the difficulties are not great.

Let the target be moving in the direction from A to B at the rate r , while the speed of the projectile is s . While the projectile moves a certain distance, the target moves r/s times this distance from a former position to the position AB . For projectiles fired from equal distances, the target, of course, moves through the same distance. The following scheme then enables us to plot very accurately the desired curve.

Construct Fig. 2 on tracing paper as follows. On the circle of Fig. 1 construct the tangents MK and LQ at M and L respectively. With M as a center and

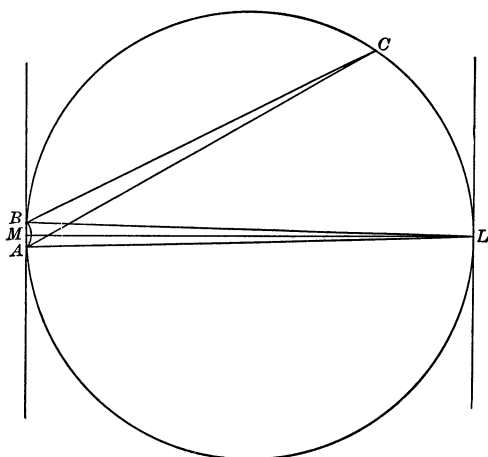


FIG. 1.

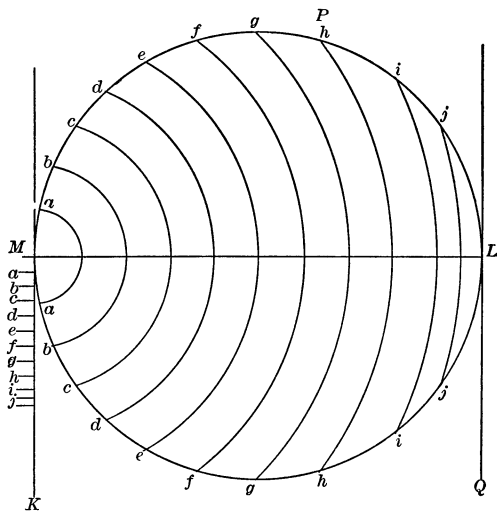


FIG. 2.

radii of lengths ranging at convenient intervals from 0 to R , strike arcs cutting the circle. From M toward K measure off distances equal to r/s times the lengths of these radii. Mark these points of division with the same symbols as the points of intersection of the corresponding arcs with the circle. On drawing paper draw two parallel lines at a distance R from each other, as OY and XV of Fig. 3. Place Fig. 2 over this drawing paper and, keeping MK and LQ coincident with OY and XV respectively, place the different division points of MK

successively over O . At each position, punch through the two papers with a pin point at the two points of the circle marked correspondingly. Draw a smooth curve (dotted in Fig. 3) through the points thus marked on the drawing paper. This is the required curve.

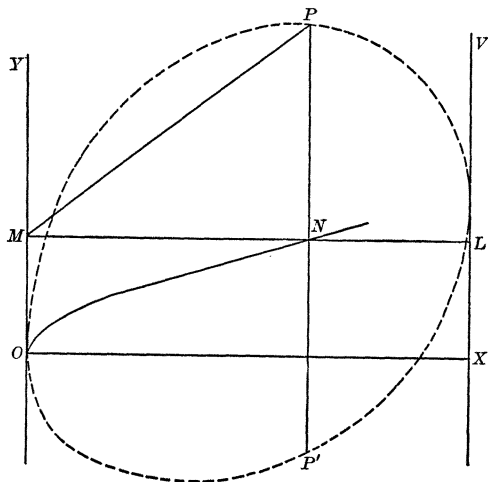


FIG. 3.

The area within the curve of Fig. 3 is referred to as the "danger area." It is evident that similar danger areas exist on both sides of AB , reflections of each other with respect to that line.

The above device is due to an English mathematician, and was used in naval strategical study during the late war. It has been interesting to work out the equations of a curve which has such an application. Results from two different simple methods are given below.

I. Let P be a point of the circle of Fig. 2 and the point determined from it in Fig. 3. Let $\angle LMP = \theta$. Then $MP = ML \cos \theta = R \cos \theta$, and

$$NP = MP \sin \theta = R \cos \theta \sin \theta.$$

Taking O as the origin, OX and OY as axes, we have

$$x = MN,$$

$$y = OM \pm NP \text{ (since } NP' = -NP\text{)}.$$

But $MN = MP \cdot \cos \theta = R \cdot \cos^2 \theta$, and $OM = r/s \cdot R \cdot \cos \theta$. Hence we have the parametric equations of the curve:

$$x = R \cos^2 \theta,$$

$$y = R \cos \theta \cdot ((r/s) + \sin \theta).$$

Whence we have

$$\cos \theta = \sqrt{x/R} = (1/R) \sqrt{Rx}, \quad \sin \theta = \pm (1/R) \sqrt{R^2 - Rx};$$

and then

$$y = (r/s) \sqrt{Rx} \pm \sqrt{Rx - x^2}$$

is the equation of the curve.

II. The curve of Fig. 3 is the locus of points of intersection of corresponding curves of two families of circles. Of one of these families, the curves all have the same radius $R/2$, but their centers move along a line parallel to the line OY (Fig. 3) and midway between lines OY and XV . The radii, ρ , of the circles of

the second family vary from 0 to R , and the centers are along OY . The centers of corresponding curves are at the same distance above O , and the radius of a circle of the second family is equal to s/r times this distance. The equations of these families are then respectively:

$$(1) \quad (x - R/2)^2 + [y - (r/s) \cdot \rho]^2 = R^2/4,$$

$$(2) \quad x^2 + [y - (r/s) \cdot \rho]^2 = \rho^2.$$

Subtracting (1) from (2) we have

$$Rx - R^2/4 = \rho^2 - R^2/4,$$

or

$$\rho^2 = Rx.$$

Substituting in (2) we obtain

$$x^2 + [y - (r/s) \sqrt{Rx}]^2 = Rx,$$

$$y - (r/s) \sqrt{Rx} = \pm \sqrt{Rx - x^2},$$

or

$$y = (r/s) \sqrt{Rx} \pm \sqrt{Rx - x^2},$$

which is the equation already obtained in I.

The general shape of this curve is evident from an inspection of this equation. The locus of the equation

$$y = (r/s) \sqrt{Rx}$$

is evidently the upper branch of the parabola

$$y^2 = (r/s)^2 \cdot Rx.$$

The quantity $\sqrt{Rx - x^2}$ is real for positive values of x less than R , and is zero for $x = 0$ or $x = R$. The danger area curve is thus seen to be a loop lying between the lines $x = 0$ and $x = R$. The points of this loop are at the distance $\sqrt{Rx - x^2}$ above and below the points (x, y) of the upper branch of the parabola $y^2 = (r/s)^2 Rx$. If the objective had been considered as moving in the opposite direction, the danger area loop would have borne the same relation to the lower branch of this same curve. The danger area on the opposite side of the objective is similarly related to the parabola $y^2 = -(r/s)^2 \cdot Rx$.